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SQUARING THE CIRCLE : DIFFUSION VOLUME AND ACOUSTIC BEHAVIOUR OF A FRACTAL STRUCTURE

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The main topic presented in this paper concerns the reduction of the diffusion volume to convex isotropic Euclidean structures as polygons and convex polyhedrons, in order to estimate the diffusive acoustic behavior of a fractal structure.

The volume V and the surface S of an Euclidean structure are defined with both quadratic and cubic relationships to the perimeter P with $S = F_q P^2$, and $V = F_C P^3$. Their geometrical nature depends on the values of the *prefactors* in those relationships, the *shape factors* F_q and F_C [1]. Extension of this relationship to the structure's generalized volume V_x [2] gives us the equation :

$$V_x = F_x P_x^D. \tag{1}$$

Calculation of the corresponding shape factors respectively gives 1/4π for a circular and 3/32π for a spherical structure. In order to generalize this expression to the whole platonician 2-D family, we operate a polygonal circle quadrature with the convexes from square to n-sided polygons, $n \to \infty$.

The quadrature factor Q represents the ratio between the circumscribed and inscribed circles of the structural polygons of the analyzed objects. They are varying from $Q = 2$ for the square (minimal quadrature) to $Q = 1$ for the circle (total quadrature). The general expression is given by :

$$Q = \frac{S_{Cir}}{S_{Ins}} = \frac{\frac{R_{Cir}^2}{2} \sin \frac{2\pi}{n}}{r_{Ins}^2 \tan \frac{\pi}{n}} \tag{2}$$

Taking into account D, fractal dimension and d, Euclidean dimension of the structure, the F_x factor expression becomes :

$$F_x = \frac{d}{2^{dQ}(d-1)2\pi^{|2-Q|}\frac{d^{Q-1}}{Q^{D-1}}} \tag{3}$$

Eq. (3) gives the F_x value of the square (F_x = 1/16), the cube (F_x = 1/64), the circle (F_x = 4π) and the sphere (F_x = 3/32π). It gives also the intermediate values of the regular polygonals which square the circle, noticing that for dodecagon, the prefactor F_x reaches the circle constant value F_x = 1/4π. This confirms the quadrature convergence.

By replacing F_x with its expression in equation (3), one can write the perimeter-surface relationship (P / S_x) for irregular 2-D shapes and to closed 2-D fractals, with its quadrature factor Q and its Euclidean dimension d as follows :

$$S_x = \frac{d}{2^{dQ}(d-1)2\pi^{|2-Q|}\frac{d^{Q-1}}{Q^{D-1}}}P^D \tag{4}$$

We are using the Minkowski analysis method to scrutinize the structure with perimeter P and real diffusion section S_x. The structuring element of radius Λ we use, gives the fractal dimension of the diffusion volume depending on perimeter P, true diffusion section S_x and the structure's fractal dimension D as follows [3] :

$$D = 1 - \frac{S_x}{P\Lambda} = 1 - \frac{\upsilon S_x}{P} \tag{5}$$

This leads to the definition domain of the *diffusion coefficient* δ which evolves [4]:

$$\delta = \frac{1}{2\upsilon\tau} = \frac{S_x(1-D)}{2P\Delta t} \tag{6}$$

where Δt represents the diffusion time-share in the structure. This leads to the diffusion time expression, depending on the fundamental length of acoustic diffusion process λ_{max}, and the roughness parameter n (number of indentations per length unit) expressed as :

$$\tau = \lambda_{max}{}^{-D} f^{d-1} = \left(F_x(1-D)\frac{P^{d(1+D-d)-1}}{n\Lambda^2} \right)^{-D} f^{d-1} \tag{7}$$

by replacing diffusion time τ with its expression in (7), the diffusion coefficient becomes :

$$\delta = \frac{1}{2\upsilon}\left(F_x(1-D)\frac{P^{d(1+D-d)-1}}{n\Lambda^2} \right)^{D} \frac{1}{f^{d-1}} \tag{8}$$

As we can see equation (8), this coefficient is defined with only the structural morphologic diffusion parameters and the sound frequency f.

That result leads us to confirm that the diffusion coefficient is a geometrical-dependant parameter, confirming the rule of the structure complexity on acoustics.

Thus, after a multiscale renormalisation, the diffusion coefficient has to be spectrally defined by adjusting a simulation procedure to experimental results, in order to implement this diffusive acoustic model for any complex configuration : those perspectives constitutes the next step of our research works.

References

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